BIOGRAPHICAL SKETCH OF PETER A. LOEB

Peter A. Loeb, born in Berkeley, California, July 3, 1937.

Present Address

Department of Mathematics, University of Illinois,
1409 West Green Street, Urbana, Illinois 61801.

Education

Reed College 1955-58; Harvey Mudd College 1958-59, B.S. in mathematics.
Princeton University 1959-1961, M.S. in mathematics.
Stanford University 1961-1964, Ph.D. in mathematics under H. L. Royden.

Academic Positions

UCLA: Instructor, 1964-65; Assistant Professor, 1965-68.
University of Illinois, Urbana-Champaign: Assistant Professor, 1968-69;
Associate Professor, 1969-75; Full Professor, 1975-2008; Now Professor Emeritus.

Visiting Positions

University of Washington: Visiting Assistant Professor, 1966-67.
Yale University: Visiting Fellow, 1971; Visiting Associate Professor, 1974.
Rutgers University: Visiting Professor, 1981.
Chalmers University of Technology, Sweden: Visiting Prof., 1988.
Cal. Tech., Pasadena: Spring 1989 and April 1996, December 2002.
Tokyo Science University and Tokyo University (JSPS Fellowship), 1989.
University of Alberta, part of fall, 2002.
National University of Singapore, month in summers 2001, 2004, 2005.

Major Invited Addresses

International Congress of Mathematicians, Warsaw, 1983.
American Mathematical Society meeting, hour address in 1987 .
Swedish Mathematical Society meeting in Göteborg, hour address in 1988.
Invited talk at Statistical Society of Canada, Calgary, May 2019.

Ph.D. Students

Paul Wake, 1971; Ye Neng Sun, 1989; Jesus Aldaz, 1991; Beate Zimmer, 1994; Vladimir Troitsky, 1999; Jesse Miller, 2011.

Editorial Boards

Logic and Analysis,
Integration: Mathematical Theory and Applications.

Awards

Japan Society for the Promotion of Science Fellowship, 1989
University of Illinois Center for Advanced Study Fellowship, 1991-92.
National Science Foundation principal investigator through summer 1998.
Fellow of the American Mathematical Society since 2013.

RESEARCH OF PETER A. LOEB

Professor Peter Loeb's research is centered in problems of representing measures and ideal boundaries in potential theory and also the application of nonstandard models to mathematical analysis. His research often uses measure spaces that are called "Loeb spaces" in the literature. Loeb and other researchers have used these methods in areas such as potential theory, stochastic processes, mathematical economics, and mathematical physics. There are many results in pure and applied mathematics, such as found in Yeneng Sun's work on the law of large numbers and Loeb's recent work with Sun on the purification of measure-valued maps, that are only valid when the underlying parameter space is a Loeb space.

Another major developed in Loeb's work is a method for showing that a limit of ratios yields a Radon-Nikodym derivative. This technique casts new light on the martingale convergence theorem and significantly simplifies the treatment of fine limit theorems in potential theory and measure differentiation theorems. An important consequence is the existence of simple boundary approach neighborhood systems in probability and potential theory that, after fixing a suitable normalization, are the "best" possible in terms of producing Radon-Nikodym derivatives as limits at the boundary. The existence of such "best" systems had not been known even for harmonic functions on the unit disk, and a result of Doob suggested that no such system could exist. An analogous construction for the differentiation of measures has also been published. Each measurable set acts as a functional on measures: The value of the functional is the measure of the set. The larger the collection of sets one has at any stage of the filtration process associated with a point, the more information one obtains when a limit exists. Given a suitable normalization, there is an "optimal", i.e. coarsest filtration process that can be used to differentiate measures. Applications include the existence of Lebesgue points and liftings of L^∞. New results on base operators and topologies are part of this research.

Related to the differentiation of measures is work on covering theorems from geometric measure theory with applications to the Henstock–Kurzweil integral of both scalar and vector valued functions. Loeb has given stronger formulations of both the Besicovitch and the Morse Covering Theorems, and together with coauthors, he has simplified the proofs. The simple proof for Besicovitch's theorem now shows that for any norm, the best associated constant (in terms of all known proofs) is a packing constant bounded above, for a d-dimensional space, by 5^d.

Recent work of Loeb with M. Insall and M. Marciniak has shown that any equivalence relation on the "remote" points of the nonstandard extension of a regular topological space produces the remainder points of a standard compactification. Moreovere, any Hausdoff compactification is produced in this way.

LIST OF PUBLICATIONS OF PETER A. LOEB

1: An axiomatic treatment of pairs of elliptic differential equations, dissertation, Stanford University, 1964.

2: A new proof of the Tychonoff Theorem, *Amer. Math. Monthly* **72**(1965) no. 7, 711-717.

3: The equivalence of Harnack's principle and Harnack's inequality in the axiomatic system of Brelot (with B. Walsh), *Ann. Inst. Fourier (Grenoble)* **15**(1965), 597-600.

4: Nuclearity in axiomatic potential theory (with B. Walsh), *Bull. Amer. Math. Soc.* **72**(1966) no. 4, 685-689.

5: An axiomatic treatment of pairs of elliptic differential equations, *Ann. Inst. Fourier (Grenoble)* **16**(1966), 167-208.

6: A minimal compactification for extending continuous functions, *Proc. Amer. Math. Soc.* **18**(1967), 282-283.

7: Sur la proportionalité de potentiels dont le support est un point polaire, *C. R. Acad. Sci. Paris* **206**(1968), 291-292.

8: An axiomatic approach to the boundary theories of Wiener and Royden (with B. Walsh), *Bull. Amer. Math. Soc.* **74**(1968), 1004-1007.

9: A maximal regular boundary for solutions of elliptic differential equations (with B. Walsh), *Ann. Inst. Fourier (Grenoble)* **18**(1968) no. 1, 283-308.

10: A criterion for the proportionality of potentials with polar point support, *Rev. Roum. Math. Pures et Appl.* **13**(1968) no. 8, 1121-1125.

11: Compactifications of Hausdorff spaces, *Proc. Amer. Math. Soc.* **22**(1969), 627-634.

12: A non-standard representation of measurable spaces and L_∞, *Bull. Amer. Math. Soc.* **77**(1971) no. 4, 540-544.

13: A non-standard representation of measurable spaces, L_∞ and L_∞^*, *Contributions to Non-Standard Analysis*, edited by W. A. J. Luxemburg and A. Robinson, North-Holland, 1972, 65-80.

14: A combinatorial analog of Lyapunov's Theorem for infinitesimally generated atomic vector measures, *Proc. Amer. Math. Soc.* **39**(1973) no. 3, 585-586.

15: A nonstandard representation of Borel measures and σ-finite measures, *Proceeding of the 1972 Victoria Conference on Nonstandard Analysis*, edited by A. E. Hurd and P. A. Loeb, Springer-Verlag Lecture Notes in Mathematics #369, 1974, 144-152.

16: A nonstandard integration theory (with A. Bernstein), *Proceedings of the 1972 Victoria Conference on Nonstandard Analysis*, edited by A. E. Hurd and P. A. Loeb, Springer-Verlag Lecture Notes in Mathematics #369, 1974, 40-49.

17: A note on continuity for Robinson's predistributions, *Proceedings of the 1972 Victoria Conference on Nonstandard Analysis*, edited by A. E. Hurd and P. A. Loeb, Springer-Verlag Lecture Notes in Mathematics #369, 1974, 153-154.

18: Editor with A.E. Hurd, *Proceedings of the 1972 Victoria Symposium on Nonstandard Analysis*, Springer-Verlag, Lecture Notes #369, 1974.

19: Conversion from nonstandard to standard measure spaces and applications in probability theory, *Trans. Amer. Math. Soc.* **211**(1975), 113-122.

20: Applications of nonstandard analysis to ideal boundaries in potential theory, *Israel J. Math.* **25**(1976), 154-187.

21: The values of nonstandard exchange economies (with Donald J. Brown), *Israel J. Math.* **25**(1976), 71-86.

22: Book review, *Non-Archimedean Fields and Asymptotic Expansions*, A. H. Lightstone and A. Robinson, *Bull. Amer. Math. Soc.* **83**(1977) no. 2, 231-235.

23: A generalization of the Riesz-Herglotz Theorem on representing measures, *Proc. Amer. Math. Soc.* **71**(1978) no. 1, 65-68.

24: An almost everywhere regular, metrizable boundary supporting the maximal representing measures for bounded and quasi-bounded harmonic functions, *Proceedings of the Third Romanian-Finnish Seminar on Complex Analysis*, Edited by Cabira Cazacu, Aurel Cornea, Martin Jurchescu and Ion Suciu, Springer-Verlag Lecture Notes in Mathematics #743, 1979, 554-563.

25: Applications of nonstandard analysis to spin models (with L. L. Helms), *J. Math. Anal. Appl.* **69**(1979) no. 2, 341-352.

26: An introduction to nonstandard analysis and hyperfinite probability theory, chapter in *Probabilistic Analysis and Related Topics,* ed. A. T. Bharucha-Reid, Academic Press, 1979, 105-142.

27: Weak limits of measures and the standard part map, *Proc. Amer. Math. Soc.*, **77**(1979) no. 1, 128-135.

28: A regular boundary supporting representing measures of bounded functions in a Bauer harmonic space, *Proceedings of 1979 Joint French and Danish Colloquium on Potential Theory*, ed. by C. Berg, G. Forst and B. Fuglede, Springer-Verlag Lecture Notes in Mathematics #787, 1980, 209-211.

29: A regular metrizable boundary for solutions of elliptic and parabolic differential equations, *Math. Annalen* **251**(1980), 43-50.

30: Book review, *Elementary Calculus* and *Foundations of Infinitesimal Calculus*, H. J. Keisler, Prindle, Weber and Schmidt, Boston 1976, *Jour. of Symbolic Logic*, **46**(1981), 673-676.

31: A nonstandard proof of the Martingale Convergence Theorem (with L. L. Helms), *Rocky Mt. Journal*, **12**(1982) no. 1, 165-170.

32: Bounds on the oscillation of spin systems (with L. L. Helms), *J. Math. Anal. Appl.* **86**(1982) no. 2, 493-502.

33: A construction of representing measures for elliptic and parabolic differential equations, *Math. Annalen* **260**(1982), 51-56.

34: A functional approach to nonstandard measure theory, in Proceedings of the S. Kakutani retirement conference, *Contemporary Mathematics*, Vol. **26**(1984), 251-261.

35: Measure spaces in nonstandard models underlying standard stochastic processes, *Proceedings 1983 International Congress of Mathematicians in Warsaw*, 323-335, PWN, Warsaw, 1984.

36: A measure-theoretic boundary limit theorem (with J. Bliedtner), *Archivs der Mathematik*, Vol. **43**(1984), 373-376.

37: A nonstandard functional approach to Fubini's Theorem, *Proc. Amer. Math. Soc.*, Vol. **93**(1985) no. 2, 343-346.

38: Book with A. E. Hurd, *An Introduction to Nonstandard Real Analysis*, Academic Press Series on Pure and Applied Mathematics, 1985.

39: Invited contribution on Lyapunov's convexity theorem (with S. Rashid) in *The New Palgrave: A Dictionary of Economic Theory and Doctrine*, MacMillan Press, London, 1987.

40: Invited contribution on nonstandard analysis (with S. Rashid) in *The New Palgrave: A Dictionary of Economic Theory and Doctrine*, MacMillan Press, London, 1987.

41: A lattice formulation of real and vector valued integrals, *Nonstandard Analysis and its Applications*, edited by Nigel Cutland, London Math. Soc. Student Texts #10, Cambridge Press, Cambridge, 1988, 221-236.

42: On semiregular points of the Martin Boundary (with A. Cornea), *Proc. Amer. Math. Soc.*, Vol. **103**(1988) no. 1, 117-124.

43: An alternative to Martin boundary theory (with J. Bliedtner), *Semesterbericht Funktionalanalysis*, Summer Semester 1988, Tübingen University, 55-60.

44: A convergence property for conditional expectation (with A. Cornea), *Annals of Probability*, Vol. **17**(1989) no. 1, 353-356.

45: On the Besicovitch Covering Theorem, *Science University of Tokyo Jour. Math.*, **25**(1989), #1, 51-55.

46: Nonstandard analysis and applications to probability theory and potential theory, *Nonstandard Analysis*, Proceedings of the meeting on nonstandard analysis held in April, 1989 at Kyoto, Japan, 1-18.

47: A note on Dixon's proof of Cauchy's integral theorem, Note in *Amer. Math. Monthly*, **98**(1991), #3, 242-244.

48: A reduction technique for limit theorems in analysis and probability theory (with J. Bliedtner), *Arkiv för Matematik*, **30**(1992), #1, 25-43.

49: Nonstandard analysis, finder of lost measures, *Analysis and Geometry, Trends in Research and Teaching*, Edited by B. Fuchssteiner

and W.A.J. Luxemburg, Wissenschaftsverlag, Mannheim, Leipzig, Wien, Zürich, 1992, pp 47-55.

50: An optimization of the Besicovitch covering, *Proc. Amer. Math. Soc., **118**(1993), #3, 715-716.*

51: A further simplification of Dixon's proof of Cauchy's integral theorem, Note in *Amer. Math. Monthly, **100**(1*993), #7, pp 680-681.

52: On the best constant for the Besicovitch covering theorem (with Z. Füredi), *Proc. Amer. Math. Soc., **121**(1994), #4, 1063-1073.*

53: The best approach for boundary limits (with J. Bliedtner), in *Classical and Modern Potential Theory and Applications*, edited by K. GowriSankaran et al., Kluwer Academic Publishers, 1994, 105-112.

54: Refining the local uniform convergence topology (with H. Osswald), in *Classical and Modern Potential Theory and Applications*, edited by K. GowriSankaran et al., Kluwer Academic Publishers, 1994, 315-316.

55: Book Review, *Nonstandard Methods in the Calculus of Variations* by Curtis Tuckey, Pitman Research Notes in Mathematics, Longman House. In *Siam Review* **37**(1995), pp459-485.

56: Best filters for the general Fatou boundary limit theorem (with J. Bliedtner), *Proc. Amer. Math. Soc., **123**(1995), #2, 459-463.*

57: Counterexamples in nonstandard measure theory (with J. M. Aldaz), Canad. Math. Bull. **38**(1995), #3, 257-261.

58: Generalized radial limits associated with representing measures (with J. Bliedtner), in *Interaction between functional analysis, harmonic analysis, and probability*, edited by N. Kalton, E. Saab, S. Montomery-Smith, Marcel Dekker Inc., 1996, 85-96.

59: Generalized Radial Limits and the Best Approach to Boundaries (with J. Bliedtner), Potential Theory-ICPT 94, Proceedings of the International Conference on Potential Theory, Kouty, Czech Republic, 1994, eds.: Král, Lukeěs, Netuka, Veselý, Walter de Gruyter (1996), 303–309.

60: Nonstandard integration theory in topological vector lattices (with H. Osswald), *Monatshefte fur Math.* **124**(1997), 53–82.

61: Opening the covering theorems of Besicovitch and Morse, *Mathematica Moravica*, Special Volume 1997, Proceedings of the 1995 International Workshop in Analysis and its Applications, 3–11.

62: Applications of nonstandard analysis to topology, *Nonstandard Analysis (Edinburgh, 1996)*, 77–89, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 493, Kluwer Acad. Publ., Dordrecht, 1997.

63: The optimal differentiation basis and liftings of L^∞ (with J. Bliedtner), *Trans. Amer. Math. Soc.* **352**(2000), 4693–4710.

64: Three chapters in and an editor (with Manfred Wolff) of the book *Nonstandard Analysis for the Working Mathematician*, Kluwer Academic Publishers (now Springer), Amsterdam, now Springer, 2000.

65: Covering theorems and Lebesgue integration (with E. Talvila), *Scientiae Mathematicae Japonicae* **53**(2001), 209–221.
66: A lost theorem of calculus, *Math. Intelligencer* **24**(2002), 15–18.
67: Nonstandard analysis and measure theory, Chapter 32 in *Handbook of Measure Theory*, E. Pap ed., Elsevier, Amsterdam, 2002, 1295–1328.
68: Sturdy harmonic functions and their integral representations, (with J. Bliedtner), *Positivity* **7**(2003), 355–387.
69: Lusin's theorem and Bochner integration (with E. Talvila), *Scientiae Mathematicae Japonicae* **60**(2004), 113–120.
70: Uncorrelatedness and orthogonality for vector-valued processes, (with H. Osswald, Y. Sun, and Z. Zhang), *Tran. .Amer. Math. Soc.* **356**(2004), 3209–3225.
71: Infinite products of infinite measures (with D. Ross), *Illinois Journal of Math.* **49**(2005), 153–158.
72: On the microscopic behavior of measurable functions, in *Nonstandard Methods and Applications in Mathematics*, ed. N.J. Cutland, M. DiNasso, and D.A. Ross, Lecture Notes in Logic 25, Association of Symbolic Logic, 2006, 123–126.
73: Purification of measure-valued maps (with Y. Sun), *Doob Memorial Volume* of the *Illinois Journal of Mathematics,* **50**(2006), 747-762.
74: Applications of rich measure spaces formed from nonstandard models, in *The Strength of Nonstandard Analysis*, ed. I. van den Berg and V. Neves, Springer-Verlag Wien NewYork, 2007, 206–210.
75: A local maximal function simplifying measure differentiation (with J. Bliedtner), *MAA Monthly,* **114**(2007), 532–536.
76: A general Fatou lemma (with Y. Sun), *Advances in Mathematics,* **213**(2007), 741–762.
77: Purification and Saturation (with Y. Sun), *Proc. Amer. Math. Soc.* **137** (2009), 2719–2724.
78: Rich measure spaces, in *Potential Theory and Stochastics in Albac, Aurel Cornea Memorial Volume*, 2009, edited by D. Bakry, L. Beznea, N. Boboc and M. Roeckner, 183–192.
79: Representing measures in potential theory and an ideal boundary, *Illinois Journal of Mathematics,* **54**(2010), 1451–1461.
80: A lost theorem of calculus, Russian translation by E. I. Gordon, *Mathematics in Higher Education*, **10**(2012), 35–43.
81: End compactifications and general compactifications (with M. Insall and M. Marciniak), *Journal of Logic and Analysis* **6:7**(2014) 1–16.
82: 4 Chapters and Editor in Second Edition *Nonstandard Analysis for the Working Mathematician*, Editors Loeb and Wolff, Springer, 2015.

83: *Real Analysis*, Birkhäuser, Switzerland, 2016: http://www.springer.com/us/book/9783319307428.
84: Chain Rule Note, to appear in *The College Mathematics Journal*.

RECENT INVITED TALKS BY PETER A. LOEB

(1) Invited hour address at AMS Meeting, Kent, Ohio, 1987.
(2) Lecture series as Visiting Professor in Sweden and Invited address to Swedish and Finnish Mathematical Societies in 1988.
(3) Three one hour talks (principal address) at meeting on Nonstandard Analysis in Kyoto, Japan, 1989.
(4) Colloquia at Tokyo University, and Universities in Chiba, Hiroshima, Kyoto, Osaka, and Nagoya in June and July of 1989 while visiting Japan on a Japan Society for the Promotion of Science Fellowship.
(5) Colloquia on nonstandard analysis in Amsterdam, Duisburg, and Konstanz, and a colloquium on covering theorems in analysis at Erlangen University, 1993.
(6) Three hour talks on nonstandard analysis in Austria, and talk at NATO meeting on potential theory in France, in 1993.
(7) Colloquia at Chalmers University of Technology, Sweden, at Universities in Bochum, Eichstätt, and Kaiserslautern in Germany, and talk in the Czech Republic during the Summer of 1994.
(8) Colloquia at Frankfurt University, Eichstätt University, Ruhr-University, and Tubingen University in Germany, in 1995–1996.
(9) Invited Lecturer, Edinburgh NATO Study Institute, 1996.
(10) Talks at the University of Illinois, Chicago, Chalmers University of Technology, Sweden, Frankfurt University and Eichstätt University, Germany, in 1997, last two also in 1998.
(11) Organizer and talk, 1999 Urbana AMS meeting special session on nonstandard analysis.
(12) Colloquia talks given in 2001 in Tuebingen and Munich Germany.
(13) Invited talks at special sessions of the AMS in Montreal and Pisa, Italy in the summer of 2002.
(14) Invited talks at the University of Alberta, Edmonton, fall 2002.
(15) Special Session, Canadian Math. Soc., in Edmonton, summer 2003.
(16) Colloquium talk at University of Hawaii, summer 2003.
(17) Invited talk at special session of the AMS on nonstandard analysis in Phoenix, Jan. 2004.
(18) Invited talk at special session of the AMS on Integration Theory in Los Angeles, April 2004.
(19) Invited talk, meeting on nonstandard analysis, Portugal, July 2004.
(20) Invited talk, Swansea, Wales, July 2004.
(21) Invited talks, National University of Singapore, summers 2001, 2004, and 2005.
(22) Invited talk, economics meeting, Purdue, October 2005.
(23) Two invited talks, meeting on nonstandard analysis, Pisa, June 2006

(24) Talks in Frankfurt Germany and Swansea Wales, May, June 2006
(25) Hour address at Osswald Retirement Colloquium, Munich, Nov.2006.
(26) Invited address at April, 2007 Conference on Modern Perspectives in Real and Stochastic Analysis, Kaiserslautern, Germany.
(27) Invited address at September, 2007 Conference on Potential Theory and Stochastics in Romania.
(28) Invited talk at meeting on nonstandard analysis, York, England, 2009.
(29) Two Invited talks at Chinese Academy of Science, Beijing, 2009.
(30) Invited talk on nonstandard analysis, University of Auckland, New Zealand, March 2014.
(31) Invited talk at Statistical Society of Canada, Calgary, May 2019.

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